CHAPTER 2

Exercise Biomechanics

Give me a lever long enough, and a prop strong enough, and I can single-handedly move the world.
-Archimedes[1]

2.1 INTRODUCTION

"Mechanics is the branch of physics concerned with the effect of forces on the motion of bodies. It was the first branch of physics that was applied successfully to living systems, primarily to understanding the principles governing the movement of animals" (Davidovits, 1975). In this chapter we are concerned with mechanical approaches to the understanding of exercise, both from a static and from a dynamic viewpoint.

Although the emphasis of this chapter is on walking, running, and moving, there also are treatments of strength, load-carrying, and muscular energy expenditure. The reader should also note that companion material can be found in Chapter 5, Thermal Responses, since an intrinsic part of biomechanical activities is the production of heat.

2.2 PHYSICS OF MOVEMENT

A great deal of understanding of movement can be obtained by consideration of fundamentals. In this section, the human body is reduced to its very simplest form, and simple conclusions result. As realistic complications are added, the analyses must become more complicated as well. However, the conclusions drawn from these involved cases will not necessarily give more insight. Thus we begin simply.

2.2.1 Equilibrium and Stability

Any body, including the human body, is in static equilibrium if the vectorial sum of both the forces and torques acting on the body is zero. Any unbalanced force results in a linear acceleration of body mass, and any unbalanced torque results in a rotational acceleration. Thus for a body to be in static equilibrium,

$$\sum \overline{F} = 0 \qquad (2.2.1a)$$

$$\sum \overline{T} = 0 \qquad (2.2.1b)$$

where $\overline{F}$ = vectorial forces, N
$\overline{T}$ = vectorial torques,[2] N·m

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[1] Lever action, torques, and forces form the basis for much of the study of biomechanics.
[2] Usually assumed to be positive for a clockwise rotation.

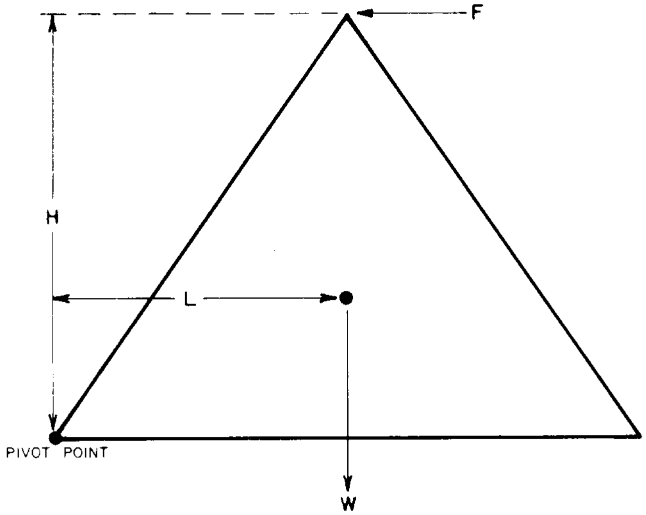

Figure 2.2.1 Body stability. The center of mass must be located over the support for stability. A wider support increases stability. Here, the action of an unbalanced force F tends to topple the body with a torque *FH.* Acting against this torque is the opposing torque WL. Increasing L increases the resistance to toppling.

TABLE 2.2.1 Fraction of Body Weights for Various Parts of the Body

Body Part	Fraction
Head and neck	0.07
Trunk	0.43
Upper arms	0.07
Forearms and hands	0.06
Thighs	0.23
Lower legs and feet	0.14
	1.00

Source: Used with permission from Davidovits, 1975

The weight of a mass can be considered to be a single force acting through a single point called the center of mass. Body weight acting through its center of mass generally is used to promote stability. That is, body weight can provide the balancing force or torque necessary to maintain stability.

The position of the center of mass with respect to the base of support determines whether the body is stable. A stable body has its center of mass directly over its support base (Figure 2.2.1). The wider the base, the more difficult it is to topple the body. The reason for this is that the lateral distance between the center of mass and the point about which the body would pivot should it topple is located at one side of the base and is increased for a wider base, producing a higher restoring torque.

The center of mass of a human body is located at approximately 56% of a person's height measured from the soles of the feet (Davidovits, 1975) and midway between the person's sides and front-to-back. The center of mass can be made to shift by extending the limbs or by bending the torso (see Table 2.2.1). When carrying an uneven load under one arm, the other arm extends from the body to compensate and shift the center of mass of the body-load

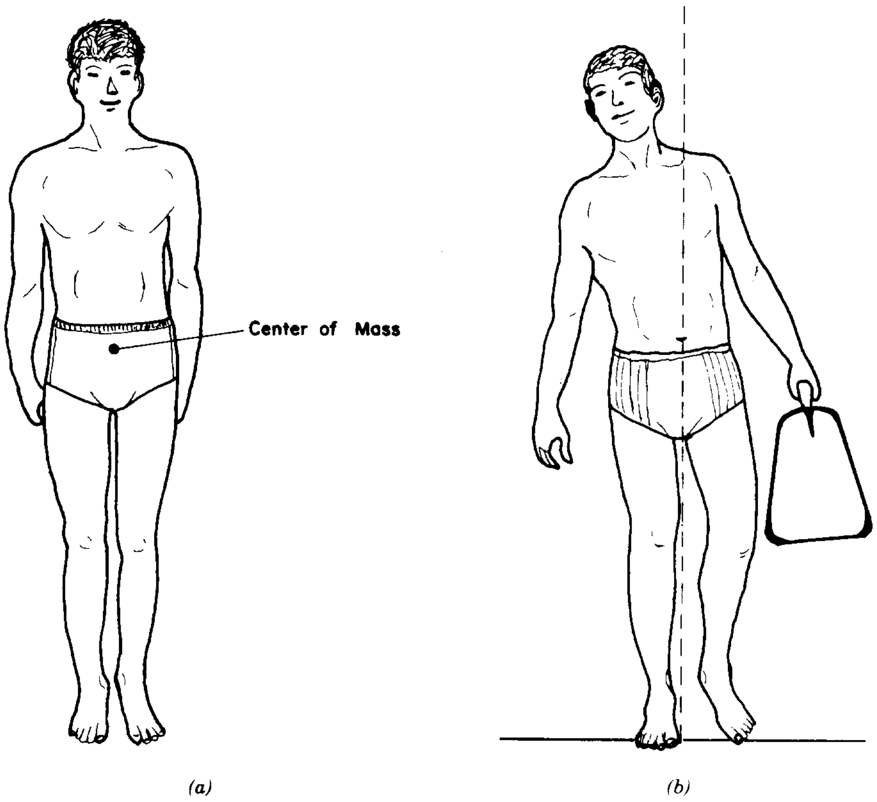

Figure 2.2.2 (a) The center of mass of the body is located at about 56% of a person's height and centered over the feet. (b) When carrying an uneven load, shifting the position of arms, legs and torso again brings the center of mass over the feet and stability is maintained. (Redrawn with permission from Davidovits, 1975.)

combination back over the feet. At the same time, the torso bends away from the load and body weight is shifted from the leg nearest the load so that the limb can help maintain stability (Figure 2.2.2).

When performing dynamic exercise, some assistance can be obtained by temporarily forcing the body to become unstable. Running, jumping, and diving are sports where instability must be managed. While wrestling, weight-lifting, and fencing, stability must be maintained. Shifting body position will produce the desired effect.

2.2.2 Muscles and Levers

Skeletal muscles consist of many thousands of parallel fibers, wrapped in a flexible sheath that narrows at both ends into tendons (Davidovits, 1975). The tendons attach the muscles to the bone. Most muscles taper to a single tendon; muscles with two tendons on one end are called biceps and muscles with three tendons are called triceps.

Muscles usually are connected between adjacent movable bones. Their function is to pull the two bones together.

Resting muscle tissue possesses an electrical potential difference across its cell membranes (Figure 2.2.3). This resting transmembrane potential arises as a consequence of the ionic charge distribution on both sides of the membrane (Mende and Cuervo, 1976). Sodium is the chief extracellular cation, and potassium is the most plentiful intracellular

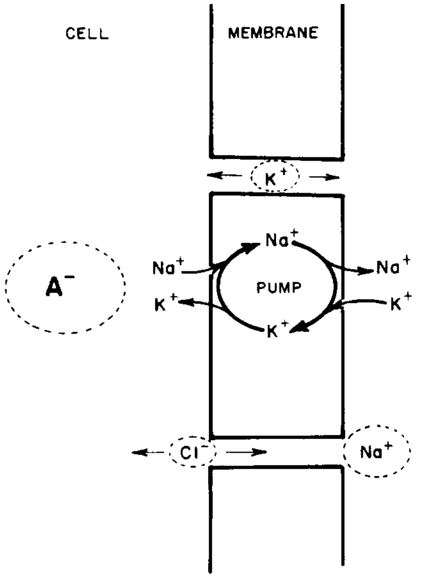

Figure 2.2.3 A transmembrane potential of - 90 mV is maintained by active mechanisms that require energy from ATP. Sodium ions are pumped outside and potassium ions are pumped inside the cell membrane. Chloride ions pass freely both ways, but large protein anions cannot escape through the cell membrance.

cation. Chloride is the main extracellular anion, whereas relatively large organic acid anions, to which the cellular membrane is impervious, are inside. A source of energy is required to establish and maintain this resting transmembrane potential. Sodium ions that leak inside the cell, due to a concentration difference across the membrane, are actively excluded. The result of this ionic disequilibrium is about a - 90 mV potential[3] difference across the membrane.

Whenever a nervous impulse reaches the muscle, a chemical transmitter is released at the site of the conjunction of nerve and muscle, which causes the muscle membrane to become much more pervious to sodium ions. The inrush of sodium actually reverses the resting transmembrane potential, and it momentarily reaches a value of about + 20 mV (+ 30 mV in neurons). Within about a millisecond, the resting value is reestablished.

This reverse polarization of the transmembrane potential travels from one location of the muscle cell to another, in wavelike fashion. Muscular contraction is triggered by this depolarization wave (see Sections 1.3.1 and 5.2.5).

Since muscles are capable only of contraction, the direction of movement of the bones to which they are attached depends on their points of attachment. In this respect, the joint between the bones acts as a fulcrum, and the muscle acts on a portion of the bone as a force on a lever.

There are three classes of levers, illustrated in Figure 2.2.4 (Davidovits, 1975). In a class 1 lever the fulcrum is located between the applied force and the load. Examples of a class 1 lever are a crowbar and a seesaw. In a class 2 lever the load is between the fulcrum and the force. A wheelbarrow is an example of a class 2 lever. In a class 3 lever, the applied force is between the fulcrum and the load. A pencil writing on a sheet of paper is an example of this class.

Equating the torques caused by the load and the applied force gives

$$Fd_F = Wd_W \tag{2.2.2}$$

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[3] The transmembrane potential is about -70mV in nerve cells and -90mV in skeletal muscle. Intracellular fluid is negative with respect to extracellular fluid.

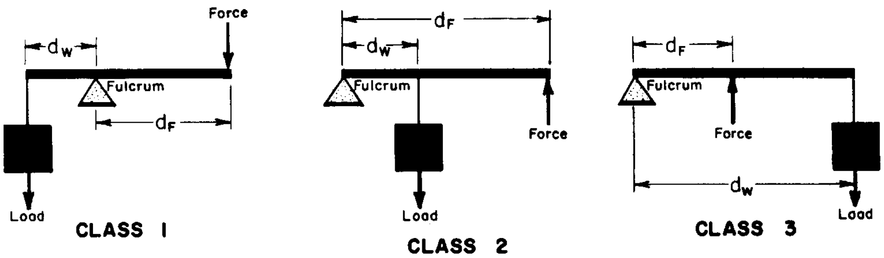


Figure 2.2.4 Three lever classes. The class 3 lever is a very common arrangement for muscles and bones. (Redrawn with permission from Davidovits, 1975.)

from which

$$\frac{F}{W}=\frac{d_W}{d_F} \tag{2.2.3}$$

where F = applied force, N
W = load, N
d_F = distance from the fulcrum to the point of application of the force, m
d_W = distance from the fulcrum to the point of attachment of the load, m

The applied force will be less than the load if the distance between fulcrum and load is less than the distance between fulcrum and the applied force. Although it may seem to be advantageous to apply a force smaller than the load, this is not the way muscular attachment is built.

Another property of levers is illustrated in Figure 2.2.5. When the load does move, the distance the load moves compared to the distance the force moves is

$$\frac{L_W}{L_F}=\frac{d_W}{d_F} \tag{2.2.4}$$

where L_W = distance through which the load moves, m
L_F = distance through which the force moves, m

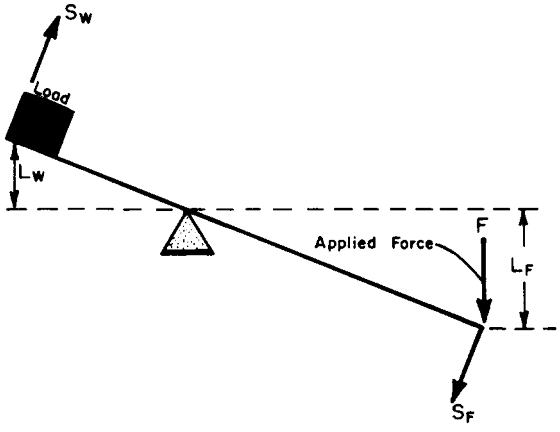


Figure 2.2.5 A class 1 lever showing the relation between distance and speed. (Redrawn with permission from Davidovits, 1975.)

When both distances are divided by time, relative speeds are obtained:

$$\frac{s_W}{s_F} = \frac{d_W}{d_F} \tag{2.2.5}$$

where s_W = speed of load movement, m/sec
s_F = speed of force movement, m/sec

Muscles are capable of generating large forces of about 7 x 10 N per square meter of cross-sectional area (Davidovits, 1975). They are not capable of moving far, and muscle efficiency decreases as speed of contraction increases (see Sections 3.2.3, 4.2.3, and 5.2.5). Therefore, many limb joints are built as class 3 levers to match the properties of muscle tissue (or vice-versa).

Figure 2.2.6 is a diagram of the upper and lower arm and elbow. The biceps muscle is attached as a class 1 lever. Calculations by Davidovits (1975) indicate that if the angle between the upper and lower arms at the elbow is about 100°, with the lower arm horizontal, then the biceps muscle exerts a force somewhat greater than 10 times the weight supported in the hand. The reaction force that is exerted by the bone of the upper arm (humerus) on the bones of the lower arm (ulna and radius) at the elbow is about 9.5 times the supported weight.

The force exerted on the joint can be significant. At the hip joint (Figure 2.2.7) the reaction force is nearly 2.5 times the weight of the person. Limping shifts the center of mass of the body more directly above the hip joint and decreases the force to about 1.25 times the body weight. This is a significant reduction in force and demonstrates why persons with injured hips limp the way they do.

Maximal expected torques that can be developed at the joints depend on several factors.

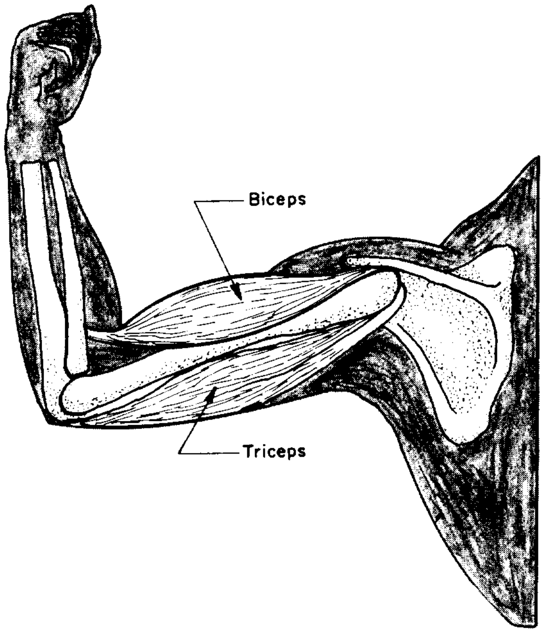
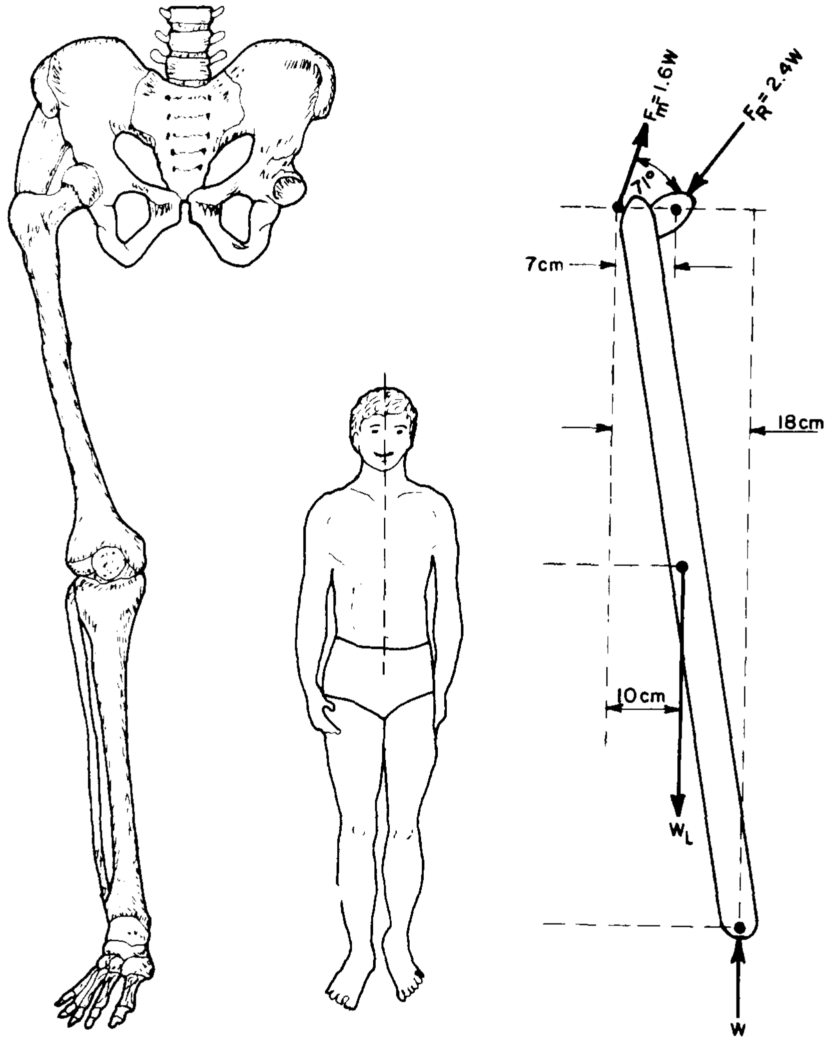


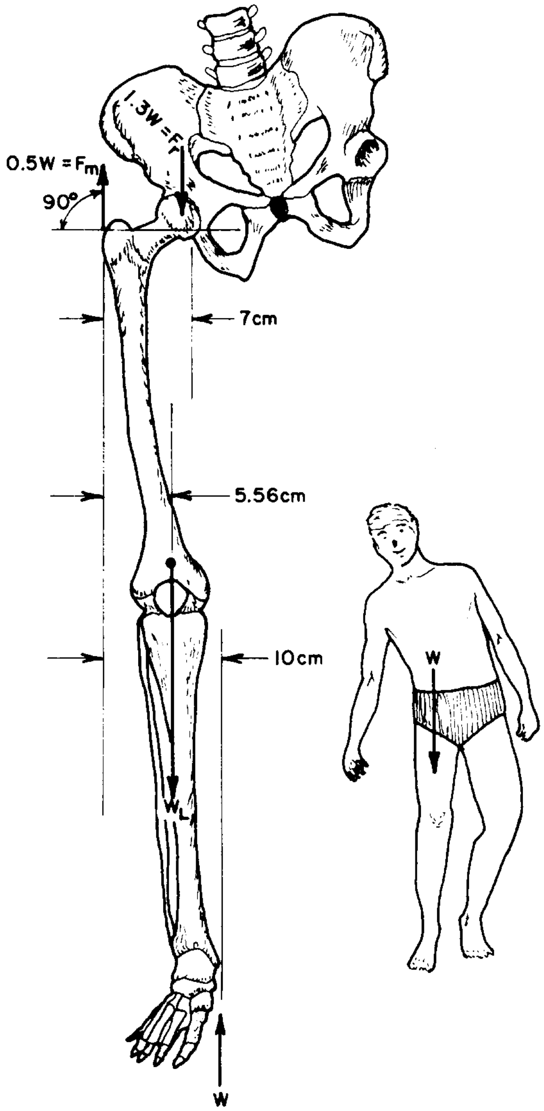
Figure 2.2.6 The muscles and bones of the elbow. The biceps muscle is attached as a class 3 lever and the triceps muscle is attached as a class 1 lever. (Redrawn with permission from Davidovits, 1975.)

First of these is the distribution of fast-twitch and slow-twitch muscle fibers in muscles of the joint (see Section 1.3.1; Kamon, 1981). Second of these is the work history of the muscles, where muscles composed largely of fast-twitch fibers can produce larger torques than muscles composed mostly of slow-twitch fibers at all speeds of contraction before muscle exhaustion. After muscle exhaustion, maximal torques are the same for the two muscle types (Kamon, 1981). Age and sex also influence maximum torque. Some of these torques are summarized in Table 2.2.2. In general, women seem to be 60% as strong as men (Kamon and Goldfuss, 1978).



(a)

Figure 2.2.7 The hip joint and reaction forces. (a) Normal posture, the hip including leg and pelvic bones, and a lever representation. Weight of the individual is designated W and the weight of the leg, W_L. Muscle force F_m is 1.6 times the body weight and the hip joint reaction force is 2.4 times the body weight. (b) Limping decreases the magnitude of both muscle force and hip reaction force on the limping side. (Redrawn with permission from Davidovits, 1975.)



(b)

Figure 2.2.7 (Continued)

2.2.3 Energy and Motion

From a general viewpoint, body motion can be considered to be composed of translational motion and rotational motion.

Translational Motion. Translational motion is characterized by identical velocity and acceleration for all parts of the body. To accelerate a body requires an unbalanced force

TABLE 2.2.2 Expected Maximal Torques[a] Around Joints Flexed at Different Angles for Average men and Women Below 40 Years of Age

		Joint Angle		
	Sex	45°	90°	135°
Shoulder flexion	Male	67	68	47
	Female	29	30	21
Elbow flexion	Male	52	85	60
	Female	24	43	23
Back extension	Male	----	240	----
	Female	----	130	----
Knee extension	Male	135	196	174
	Female	93	130	136
Foot plantar flexion	Male	110	127	101
	Female	83	111	108

Source: Used with permission from Kamon, 1981
[a] All torque values in N·m

according to the familiar Newton's second law:

$$F=\frac{d(mv)}{dt}=m\frac{dv}{dt}=ma \tag{2.2.6}$$

where mv = translational momentum of a body, kg·m/sec
m = body mass, kg
v = body velocity, m/sec
F = force, N
t = time, sec
a = acceleration, m/sec^2

A body that is subjected to uniform acceleration for a time t will reach a speed[4]

$$s = s_0 + at \tag{2.2.7}$$

where s = speed, m/sec
s_0 = initial speed, m/sec

Integration of Equation 2.2.7 will give distance traveled over that time:

$$L=\int_0^t s\,dt=\int_0^t (s_0+at)dt=s_0t+\frac{at^2}{2} \tag{2.2.8}$$

where L = distance traveled, m

Energy is defined as the capacity of a body to do work. Kinetic energy is the result of motion and potential energy is the result of position. For calculation of energy, a force times the distance through which it acts is required:

$$E = FL \tag{2.2.9}$$

where E = energy, N·m

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[4] The difference between velocity and speed is that the former is a vector quantity (includes a direction and a magnitude) whereas the latter is a scalar quantity (magnitude only).

For kinetic energy, assuming zero initial velocity and uniform acceleration,

$$F = ma \tag{2.2.10a}$$

$$L = \frac{at^2}{2} \tag{2.2.10b}$$

$$v = at \tag{2.2.10c}$$

$$E = (ma)\left(\frac{at^2}{2}\right) = \frac{m}{2}(at)^2 = \frac{mv^2}{2} \tag{2.2.10d}$$

For potential energy within a constant gravitational field,

$$F = mg \tag{2.2.11a}$$

$$L = h \tag{2.2.11b}$$

$$E = mgh \tag{2.2.11c}$$

where h = height above a reference plane, m

g = acceleration due to gravity, 9.80 m/sec^2

Vertical jumps begin from a crouch (Figure 2.2.8). The legs push against the bottom surface until the feet leave the surface, and the body continues to rise until decelerated to zero velocity by gravity. The maximum height of the jump is the point when velocity reaches zero. Energy performed by the legs to raise the body from the crouch (kinetic energy) is translated into potential energy in the process of the jump. From a conservation of energy perspective,

$$Fc = W(c + h) \tag{2.2.12}$$

where F = force produced by the legs, N

W = body weight, N

c = depth of the crouch, m

h = height of the jump, m

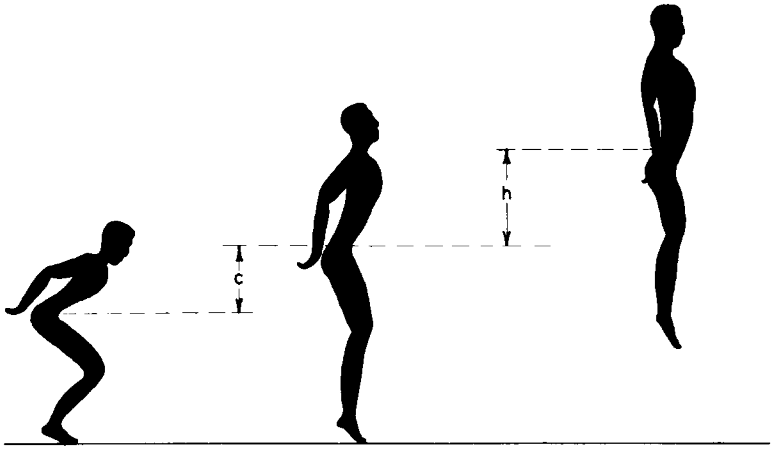

Figure 2.2.8 Vertical jump. Crouching before the jump gives the legs opportunity to develop more jumping energy than if the jump began from a standing position. (Redrawn with permission from Davidovits, 1975.)

Therefore,

$$h=\frac{(F-W)c}{W} \tag{2.2.13}$$

Experimental measurements have shown that the force produced by the legs is roughly twice the body weight (Davidovits, 1975). Thus

$$h = c \tag{2.2.14}$$

For an average person, the depth of the crouch is about 60 cm. The height of the jump is also about 60 cm.

A much greater height ought to be attained by beginning the jump from a running start. Horizontal kinetic energy can be converted into vertical potential energy, thus forming the basis for an estimate of the height of the jump:

$$mgh=\frac{1}{2}mv^2 \tag{2.2.15a}$$

$$h=\frac{v^2}{2g} \tag{2.2.15b}$$

Added to this estimate should be a considerable fraction of the 60 cm previously estimated as the height boost that can be produced in a final pushoff by the legs just before jumping. Also, note that the center of mass of the body is about a meter above the feet, and that by repositioning the body during the high jump to a more nearly horizontal plane (with a net external energy cost of very little because, as the lower body is being raised, the upper body is lowered), a higher level can be cleared. We must subtract from our estimate a very small amount of translational kinetic energy which cannot be converted into potential energy because it is needed to jump over the bar. Therefore,

$$h=\frac{v^2}{2g}+1.4 \tag{2.2.15c}$$

The short-distance running speed of a good high jumper is about 8.2 m/sec (Davidovits, 1975). The estimate of height thus becomes 4.8 m.

This estimate for the high jump is about twice the high jump record. Furthermore, the pole vault recorded is near 6.0 m. These facts demonstrate that the efficiency of transforming translational kinetic energy into potential energy is much higher with the aid of a pole than with the unaided foot.

If the vertical jump is performed in a weaker gravitational system, such as on the moon, a greater height can be attained. However, the additional height is not proportional to the decrease in weight. Because of the gravitational system, the maximum force produced by the legs does not change, nor does the depth of the crouch change. Return to Equation 2.2.13:

$$\frac{h_m}{h}=\frac{(F-W_m)}{(F-W)}\frac{W}{W_m}=\frac{(2W-W_m)}{(2W-W)}\frac{W}{W_m} \tag{2.2.16}$$

where h_m = height of jump on the moon, m

W_m = weight of person on the moon, N

With one-sixth of the earth's gravity, the moon causes a person to weigh one-sixth what he would on the earth ($W/W_m = 6$). Therefore, a person who jumps 60 cm on the earth will jump 6.6 m on the moon.

Energy considerations can also be used to calculate the height of a jump that will produce bone fracture. When assuming bone to be an elastic material, the energy stored in this elastic material is

$$E = \frac{1}{2} K (\Delta L)^2 \tag{2.2.17}$$

where K = spring constant, N/m
ΔL = change in length from resting length, m

The spring constant is a property of an elastic material (analogous to a spring) which relates force required to compress the spring to the compression distance:

$$F = K\Delta L \tag{2.2.18}$$

Stress is defined as the force in a material divided by the cross-sectional area:

$$\sigma = F/A \tag{2.2.19}$$

where σ = stress, N/m^2
F = force, N
A = cross-sectional area, m^2

and strain is defined as the amount of compression or stretch divided by the original length:

$$\varepsilon = \Delta L/L \tag{2.2.20}$$

where ε = strain, m/m
L = original material length, m
ΔL = change in length, m

The ratio of stress to strain, called Young's modulus (also called modulus of elasticity or elastic modulus), is usually assumed to be constant[5] and has been measured for many materials:

$$Y = \frac{\sigma}{\varepsilon} \tag{2.2.21}$$

where Y = Young's modulus, N/m^2

Young's modulus for bone in compression is 1.4×10^{10} N/m^2 (Davidovits, 1975).

Also measures is the maximum compressive stress that can be resisted without rupture. For bone, this value is 10^8N/m^2 (Davidovits, 1975).

Combining Equations 2.2.18 through 2.2.21,

$$K = \frac{F}{\Delta L} = \frac{F/A}{\Delta L/A} = \frac{\sigma A}{\Delta L} = \frac{\sigma A/L}{\varepsilon} = \frac{YA}{L} \tag{2.2.22}$$

At the maximum compressive stress,

$$Y = \frac{\sigma_{\max}}{\varepsilon} = \frac{\sigma_{\max}}{\Delta L/L} \tag{2.2.23a}$$

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[5] For many biological materials, the ratio of stress to strain is not truly constant, usually becoming lower at higher rates of strain. In this case, Young's modulus is often measured as the slope of the chord joining the origin to a point on the curve with a particular strain.

and

$$\Delta L = \frac{\sigma_{\max} L}{Y} \tag{2.2.23b}$$

where $\sigma_{\max}$ = maximum breaking stress, N/m^2

An energy balance can now be written for the leg bones in compression. The energy input is the body weight times the height of the fall. Energy stored in the bone is given by Equation 2.2.17:

$$Wh = \frac{1}{2} K (\Delta L)^2 = \frac{AL\sigma_{\max}^2}{2Y} \tag{2.2.24a}$$

$$h = \frac{AL\sigma_{\max}^2}{2YW} \tag{2.2.24b}$$

where h = height of the fall, m
A = total cross-sectional area of the bones of the legs, m^2
L = length of the leg bones, m
W = body weight, N
$\sigma_{\max}$ = maximum breaking stress, N/m^2

Taking the combined length of the leg bones at about 90 cm and the combined area of the bones in both legs at about 12 cm^2, and assuming an average 686 N body weight (70 kg mass), the allowable height of the jump is 56 cm. Obviously, jump heights greater than 56 cm are safely made. But this does point to the fact that a great deal of energy is dissipated in bone joints and in the redistribution of fall energy on landing.

Not only do the joints aid in protecting the bones from breaking, but they also possess an amazing amount of lubrication, which keeps them from destruction. Since the center of mass is not directly above the hip joint, the force exerted by the bones on the joints is about 2.4 times the body weight (see Figure 2.2.7*a*). The joint slides about 3 cm (0.03m) inside the socket during each step. The friction force acting through this distance is the coefficient of friction times the exerted force, or $2.4 \cdot \mu \cdot W$. The energy expanded during each step is

$$E = FL = (2.4W)(\mu)(0.03) \tag{2.2.25}$$

where μ = friction coefficient, dimensionless

Without lubrication, the coefficient of friction would be about 0.3 and the energy to be dissipated during each step of a 686 N man would be nearly 15 N·m; the joint would be destroyed. As it is, the joint is well lubricated and has a coefficient of friction of only 0.003, reducing friction heat and wear to negligible values.

Angular Motion. Any object moving along a curved path at a constant angular velocity is subject to a centrifugal force:

$$F_c = \frac{mv^2}{r} = \frac{Wv^2}{gr} \tag{2.2.26}$$

where F_c = centrifugal force, N
m = body mass, kg

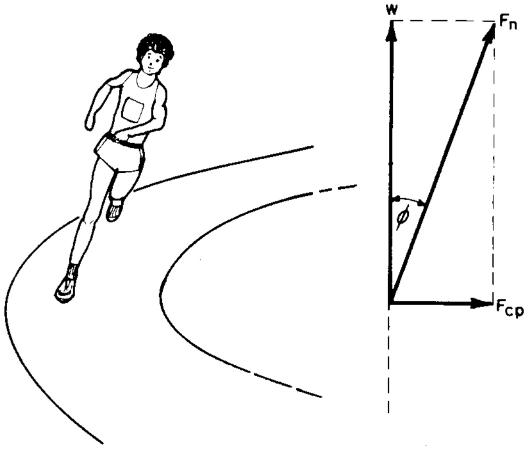

Figure 2.2.9 Runner on a curved track and a representation of the forces acting on the foot of the runner. (Redrawn with permission from Davidovits, 1975.)

v = velocity[6] of the body tangential to the curve of the path taken by the body, m/sec
r = radius of curvature, m
W= body weight, N
g = acceleration due to gravity, 9.8 m/sec^2

This centrifugal force component must be balanced by a force of equal magnitude and opposite direction, called the centripetal force, in order that the body does not slide radially outward from the curve. Centripetal force may be supplied by friction:

$$F_{\mathrm{cp}} = \mu W = \frac{Wv^2}{gr} \tag{2.2.27}$$

where F_{cp} = centripetal force, N
μ = coefficient of friction, dimensionless

or it may be supplied on a banked curve by the component of force acting toward the center of the curve (Figure 2.2.9):

$$F_{\mathrm{cp}} = F_n \sin\phi = \frac{Wv^2}{gr} \tag{2.2.28}$$

where F_n = force normal to the surface of the banked curve, N
ϕ = angle of the curve with respect to the horizontal, rad

Since the vertical component of F_n must support the weight of the body,

$$F_n \cos\phi = W \tag{2.2.29a}$$

$$F_n = \frac{\cos\phi}{W} \tag{2.2.29b}$$

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[6] The linear distance traversed in angular motion is

$$D = r\theta$$

where D = distance, m
r = radius of curvature, m
θ = angle of the curve traversed, rad

Dividing both sides of the equation by time gives

$$\frac{D}{t} = v = r\frac{\theta}{t} = r\omega$$

where ω = angular velocity, rad/sec.

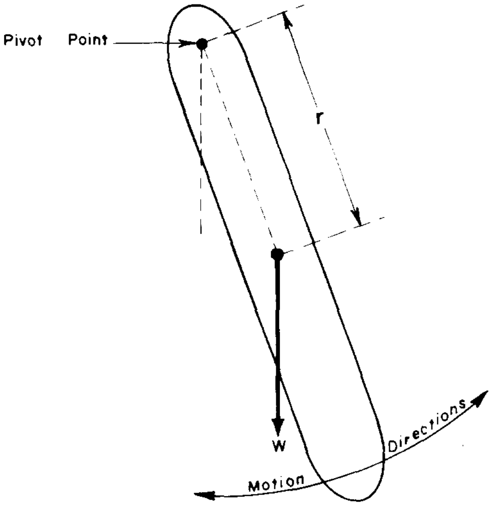

Figure 2.2.10 Diagram of a physical pendulum. (Redrawn with permission from Davidovits, 1975.)

Then, without friction,

$$tan\phi=\frac{sin\phi}{cos\phi}=\frac{v^2}{gr} \qquad (2.2.30)$$

The only way that any given banking angle can support various running speeds is by friction to supply the otherwise unbalanced centrifugal force.

A runner rounding a curve, as in Figure 2.2.9, naturally leans into the curve. The reason for this is that the resultant force F_n will pass through the center of mass of the body only if the runner leans inward. If not, there will be an unbalanced torque acting on the body which tends to topple the runner outward. The angle of the lean is the same as calculated by Equation 2.2.30. For the speed of 6.7 m/sec (a 4 min mile) on a 15 m radius track, $\phi = 0.30$ rad (17°). Notice that body weight does not influence this angle. Also to be noted is the fact that the banking on running tracks must be tailored to the speeds expected to be run on them.

For simplistic analysis of walking, consider the legs as pendulums. However, simple pendulums, with all the weight concentrated at the ends, are not a good representation of the legs. The physical pendulum is more realistic because its weight is distributed along its length (Figure 2.2.10).

The period of oscillation for a physical pendulum (Davidovits, 1975) is

$$T=2\pi\sqrt{\frac{1}{Wr}} \qquad (2.2.31)$$

where T = periods of oscillation, sec
I = moment of inertia, N·m·sec^2
r = distance from pivot point to center of mass, m
W = weight of the pendulum, N

and

$$I=\frac{WL^2}{3g} \qquad (2.2.32)$$

where L = length of the pendulum, m
g = acceleration of gravity, 9.8 m/sec^2
if the center of mass of a leg can be assumed to be at half its length, then

$$T = 2\pi\sqrt{\left(\frac{2}{3}\frac{L}{g}\right)} \tag{2.2.33}$$

For a 90 cm long leg, the period is 1.6 sec (Davidovits, 1975). If each walking step is regarded as a half-swing (the time of the pendulum to swing forward), then the time for each step is $T/2$. This is the most effortless walk; walking faster or slower requires additional muscular exertion and is more tiring.

Walking speed is proportional to the number of steps in a given time, and the size of each step is proportional to the length of the leg. Therefore,

$$s \propto \frac{L}{T} \tag{2.2.34a}$$

where s = walking speed, m/sec
But, from Equation 2.2.33,

$$T \propto \sqrt{L} \tag{2.2.34b}$$

Therefore,

$$s \propto \frac{L}{\sqrt{L}} = \sqrt{L} \tag{2.2.34c}$$

Thus the speed of walking in a natural stride increases as the square root of the length of the walker's legs. Similarly, the natural walking speeds of smaller animals are slower than those of larger animals.

The situation for running, however, is different. When running, the torque is produced mostly by the muscles instead of gravity. Assume that the length of the leg muscles is proportional to the length of the leg, the cross-sectional area of the muscles is proportional to the length squared, and the mass of the leg is proportional to length cubed[7]:

$$L_m \propto L \tag{2.2.35a}$$

$$A_m \propto L^2 \tag{2.2.35b}$$

$$m \propto L^3 \tag{2.2.35c}$$

where L_m = muscle length, m
A_m = muscle area, m^2
m = leg mass, kg

Maximum muscle force is proportional to the area of the muscle. Maximum muscle torque is proportional to the product of the maximum force times the length of the leg:

$$T_{\max} \propto F_m L \propto L^3 \tag{2.2.36}$$

---

[7] Leg mass proportional to length cubed implies that body mass is proportional to its length cubed. Although we like to think this is true, a least squares regression of ideal body weights, as published by the American Heart Association, with height for medium-frame men, gives a dependence of mass on height to the 1.4 power.

where $T_{\max}$ = maximum muscle torque, N·m

F_m = maximum muscle force, N

The period of oscillation for a physical pendulum with application of an external torque (Davidovits, 1975) is

$$T = 2\pi\sqrt{\frac{I}{T}} \tag{2.2.37}$$

With the mass of the leg proportional to L^3, the moment of inertia becomes proportional to L^5. Therefore, the period of oscillation becomes

$$T \propto \sqrt{\frac{L^5}{L^3}} = L \tag{2.2.38}$$

Running speed is still proportional to the product of the number of steps per unit time and the length of each step. Therefore,

$$s \propto \frac{L}{T} \propto \frac{L}{L} = 1 \tag{2.2.39}$$

This indicates that the maximum speed of running is independent of leg size. A fox, for instance, can run at about the same speed as a horse (Davidovits, 1975).

This simple analysis, which we will see later needs considerable modification to reflect reality, can be used to give an estimate of the energy expended during running. The legs are assumed to pivot only at the hips and reach their maximum angular velocity as the feet swing past the vertical position. Rotational kinetic energy at this point (Davidovits, 1975) is

$$E_r = 1/2\, I\, \omega^2 \tag{2.2.40}$$

where E_r = rotational kinetic energy, N·m

ω = angular velocity, rad/sec

and this energy is assumed to be supplied by the leg muscles during each running step. The angular velocity can be calculated (Davidovits, 1975) from

$$\omega = \frac{s_{\max}}{L} \tag{2.2.41}$$

where $s_{\max}$ = leg speed with the leg in the vertical position, m/sec

= speed of running

Energy calculated using Equation 2.2.40 must be divided by muscular efficiency (about 20%) to obtain total energy expenditure. Using some very simplifying assumptions, Davidovits (1975) calculated the energy of running for a 70 kg person with 90 cm long legs with 90 cm step lengths to be 100 kN·m (24 kcal) when running 1.6 km (1 mile) in 360 see (6 min). This compares to a value of 1352 N·m/sec (19.4 kcal/min) energy expenditure from Table 2.3.1. The conclusion of this exercise is that there is a good deal more to calculating the energy of running than given in this simple example.

2.3 THE ENERGY COST OF MOVEMENT

We all know that various types of movement require different energy levels. From the data of Table 2.3.1 we can see that the energy contained in a large apple can be expended by 19 min

TABLE 2.3.1 Calorie–Activity Table: Energy Equivalents of Food Calories, Expressed in Minutes of Physical Activity

			Activity				
	Energy,						
Food	kN·m	(kcal)	Walking[a]	Riding Bicycle[b]	Swimming[c]	Running[d]	Reclining[e]
Apple, large	423	(101)	19	12	9	5	78
Bacon, 2 strips	402	(96)	18	12	9	5	74
Banana, small	368	(88)	17	11	8	4	68
Beans, green, 1 c	113	(27)	5	3	2	1	21
Beer, 1 glass	477	(114)	22	14	10	6	88
Bread and butter	327	(78)	15	10	7	4	60
Cake, two-layer, 1/12	1490	(356)	68	43	32	18	274
Carbonated beverage, 1 glass	444	(106)	20	13	9	5	82
Carrot, raw	176	(42)	8	5	4	2	32
Cereal, dry, 1/2 c, with milk and sugar	837	(200)	38	24	18	10	154
Cheese cheddar 1 oz	465	(111)	21	14	10	6	85
Cheese, cottage, 1 tbsp	113	(27)	5	3	2	1	21
Chicken, fried, ½ breast	971	(232)	45	28	21	12	178
Chicken, TV dinner	2270	(542)	104	66	48	28	417
Cookie, chocolate chip	213	(51)	10	6	5	3	39
Cookie, plain	63	(15)	3	2	1	1	12
Doughnut	632	(151)	29	18	13	8	116
Egg, boiled	322	(77)	15	9	7	4	59
Egg, fried	460	(110)	21	13	10	6	85
French dressing, 1 tbsp	247	(59)	11	7	5	3	45
Gelatin, with cream	490	(117)	23	14	10	6	90
Halibut steak, 1/4 1b	858	(205)	39	25	18	11	158
Ham, 2 slices	699	(167)	32	20	15	9	128
Ice cream, 1/6 qt	808	(193)	37	24	17	10	148
Ice cream soda	1070	(255)	49	31	23	13	196

Ice milk, 1/6 qt	603 (144)	28	18	13	7	111
Malted milk shake	2100 (502)	97	61	45	26	386
Mayonnaise, 1 tbsp	385 (92)	18	11	8	5	71
Milk, 1 glass	695 (166)	32	20	15	9	128
Milk, skim, 1 glass	335 (81)	16	10	7	4	62
Milk shake	1760 (421)	81	51	38	22	324
Orange, medium	285 (68)	13	8	6	4	52
Orange juice, 1 glass	502 (120)	23	15	11	6	92
Pancake with syrup	519 (124)	24	15	11	6	95
Peach, medium	193 (46)	9	6	4	2	35
Peas, green, ½ c	234 (56)	11	7	5	3	43
Pie, apple, 1/6	1580 (377)	73	46	34	19	290
Pie, raisin, 1/6	1830 (437)	84	53	39	23	336
Pizza, cheese, 1/8	753 (180)	35	22	16	9	138
Pork chop, loin	1310 (314)	60	38	28	16	242
Potato chips, 1 serving	452 (108)	21	13	10	6	83
Sandwiches						
Club	2470 (590)	113	72	53	30	454
Hamburger	1460 (350)	67	43	31	18	269
Roast beef with gravy	1800 (430)	83	52	38	22	331
Tuna fish salad	1160 (278)	53	34	25	14	214
Sherbet, 1/6 qt	741 (177)	34	22	16	9	136
Shrimp, French fried	753 (180)	35	22	16	9	138
Spaghetti, 1 serving	1660 (396)	76	48	35	20	305
Steak, T-bone	984 (235)	45	29	21	12	181
Strawberry shortcake	1670 (400)	77	49	36	21	308

[a] Energy cost of walking for 686 N (70 kg) individual = 363 N·m/sec (5.2 kcal/min) at 1.56m/sec (3.51 mi/hr).

[b] Energy cost of riding bicycle = 572 N·m/sec (8.2 kcal/min).

[c] Energy cost of swimming = 781 N·m/sec (11.2 kcal/min).

[d] Energy cost of running = 1353 N·m/see (19.4 kcal/min).

[e] Energy cost of reclining = 90.7 N·m/sec (1.3 kcal/min).

of walking, 12 min of cycling, 9 min of swimming, 5 min of running, and by 78 min of reclining. This indicates that running is the most energy-intensive exercise among the five. However, with that same energy expenditure, a walker will cover a distance of about 1.8 km, the cyclist will cover a distance of about 4.8 km, the swimmer will go only 360 m, the runner will travel 2.0 km, and the recliner will not travel at all. Clearly, there is a huge difference between the energy expended on these different tasks. Why this should be so is the topic of this discussion.

The case of the cyclist is most interesting. The cyclist encumbers himself with the extra weight of the apparatus, but he obviously, gains a great deal by being able to travel substantially farther on the same amount of energy compared to walking or running (which have nearly equal distances). We previously noted the special case of swimming (Section 1.2) and the additional energy required to overcome viscous drag on the body. Returning to the case of bicycling, what is it about the bicycle that makes locomotion with it so highly efficient?

Tucker (1975) considered this and other forms of movement. In his article he proposed, as an index of the cost of transport,

$$\mathrm{CT} = P_i / sW \tag{2.3.1}$$

where CT = cost of transport, dimensionless
P_i = input power, N·m/sec
s = speed of movement, m/sec
W = body weight, N

The cost of transport really involves the rate of energy usage moving at an appropriate speed. Because there may be substantial differences in body weight between animals to be compared,[8] the cost of transport includes the weight factor. The result is a dimensionless quantity that can be used to compare different modes of exercise.

The cost of transport for a given animal will vary with speed. If the animal does not move, the cost of transport will be infinite because speed is zero, but a small amount of maintenance energy (see Section 5.2.5) is still supplied. At very rapid speeds, the energy cost is very high due to friction and inertia of various body parts. In Section 2.2.3 we saw that walking and running speeds could be related to the natural periods of pendulums. Faster speeds require the use of additional forcing energy. Thus at very high speeds, as at low speeds, the cost of transport becomes very high. In between, there will be a minimum power expenditure at some point.

The cost of transport will also achieve a minimum, but generally at a higher speed than the power expenditure minimum. For a constant animal weight, the minimum cost of transport will be determined by the ratio of P_i/s, which is equivalent to determining the minimum graphically as the point at which a line through the origin of a graph for P_i and s is tangent to the power curve. Figure 2.3.1 shows this minimum for flight of a budgie (budgerigar parrot).

In Figure 2.3.2 are plotted minimum costs of transport for a variety of runners, fliers, swimmers, and other forms of human locomotion. Over 12 orders of magnitude of body mass are represented, and minimum costs of transport vary widely.

The data in Figure 2.3.2 cluster along three general lines of classification: swimmers, fliers, and walkers (or runners), with the minimum costs of transport for swimmers less than those for fliers and for fliers less than those for walkers. Walking, therefore, is a comparatively inefficient way of moving about.

Cycling has a minimum cost of transport about one-fourth that of walking,[9] which is why a cyclist is willing to assume the burden of the extra weight of the bicycle. Human swimmers, on the other hand, have a minimum cost of transport nearly six times that of human walkers.

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[8] Or the body weight of a given animal may change substantially over a short time. Some migrating birds use up to 25% of their body weight as fuel between feeding periods (Tucker, 1975).

[9] A 686 N man (mass of 70 kg) achieves his minimum cost of transport while walking at about 1.75 m/sec (3.85 mi/hr). The metabolic cost of walking at this speed is 452 N·m/sec and his cost of transport is 0.376. By comparison, he expends 1122 N·m/sec while jogging at 3.5 m/sec (7.7 mi/hr) and his cost of transport is 0.467.

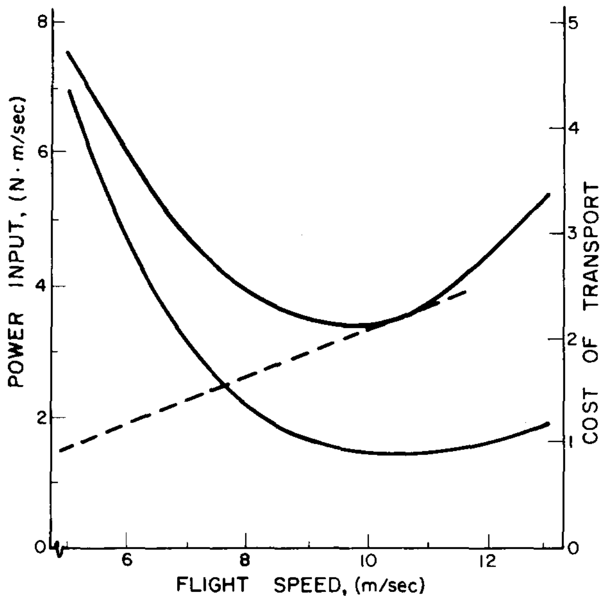

Figure 2.3.1 The cost of transport (upper curve) and power input (lower curve) for a 0.35 kg parrot in level flight. There is a minimum power input at a speed between 10 and 11 m/sec. The minimum cost of transport is found at the point of tangency of the curve and the dashed line. (Redrawn with permission from Tucker, 1975.)

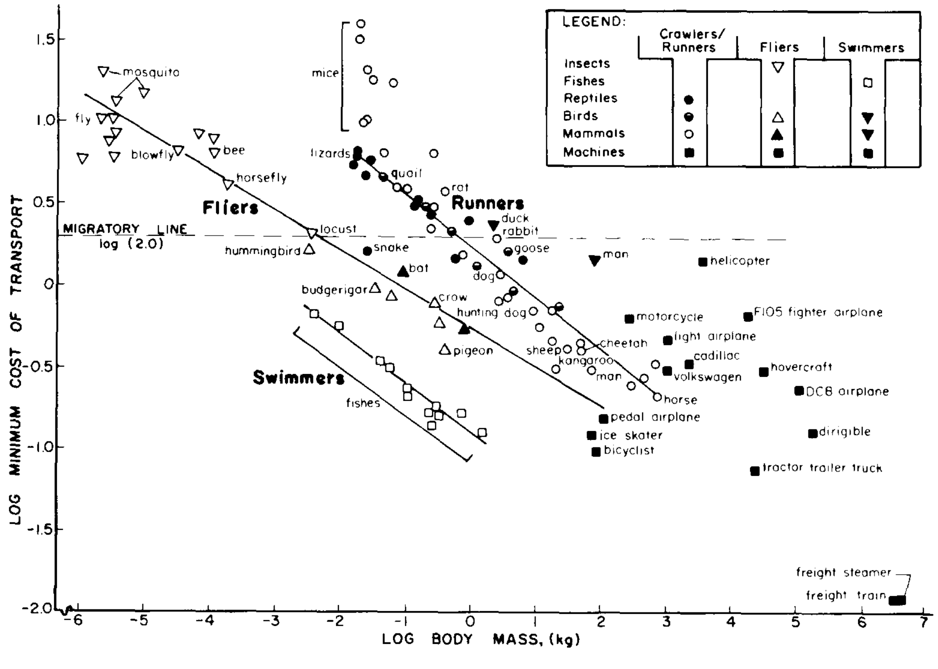

Figure 2.3.2 Minimum costs of transport for various species, which fall naturally into groups depending on their types of locomotion. (Redrawn with permission from Tucker, 1975.)

Many cars and airplanes have costs of transport worse than walking animals of equivalent mass, but tractor trailer trucks are nearly as efficient as walking animals of equivalent mass (if they existed!).

A sparrow possesses a mass and metabolic rate equivalent to a mouse but flies nearly 10 times faster than the mouse runs. The sparrow's minimum cost of transport is about 10 times less than that for a mouse.

Tucker (1975) observed that smaller terrestrial animals almost never migrate, but smaller birds often do. Only larger mammal species, such as caribou, bison, and large antelopes, migrate. Figure 2.3.2 shows a horizontal line at a cost of transport of 2.0. Animals with lower costs of transport have usually been observed as migratory species, whereas animals above the line have not. Apparently the costs of transport are too high for migration if they are above 2.0.[10]

The statement of muscular efficiency, at least for the larger muscles, being about 20%, is made several times in this book (Sections 1.3, 2.2, 3.2.3, 4.2.3, 5.2.5). However, considering an act of movement as a whole, mean muscular efficiency often is much lower than this and may approach zero. Such is the case with walking on a level surface.

Muscle power is used, in general, for three purposes: (1) to support the body weight, (2) to overcome aerodynamic drag, and (3) to perform mechanical work. Total input power equals muscle power plus power diverted for nonmuscular purposes:

$$P_i - P_{\mathrm{nm}} = \frac{P_{\mathrm{spt}} + P_d + P_w}{\eta} \tag{2.3.2}$$

where P_i = input power, N·m/sec
P_{nm} = nonmuscular power, N·m/sec
P_{spt} = power to support body weight, N·m/sec
P_d = power to overcome drag, N·m/sec
P_w = power to perform external mechanical work, N·m/sec
η = muscular efficiency, dimensionless

Rearranging Equation 2.3.2 to obtain mean muscular efficiency:

$$\eta = \frac{P_{\mathrm{spt}} + P_d + P_w}{P_i - P_{\mathrm{nm}}} \tag{2.3.3}$$

For walkers or runners, the power required to support the body weight is very small. So is aerodynamic drag. Since a walker or runner on the level does not raise his body weight,[11] external work is zero. Therefore, mean muscular efficiency for walking and running approaches zero.

Birds do not have the same efficiencies while flying. They must support their body weights with their wing muscles; going faster, they have higher amounts of aerodynamic drag; and they perform external work when they move their wings through the air. Their mean muscular efficiencies are close to 20%.

Why should mean muscular efficiency of walking be so low, and what happens to the input energy? While walking, the center of mass of the body is continually moving up and down. The muscles actively perform external work to raise the body weight, but they cannot recover the potential energy when the center of mass falls. Instead, the muscles act against the body weight by decelerating it. When muscles shorten and produce a force during shortening, they produce external work; when muscles stretch but produce a force against an externally

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[10] Notice that helicopters and F105 fighter planes could migrate, should they be so inclined.

[11] We are talking here about raising or lowering body weight over the entire walking or running cycle. During the cycle, however, body weight does rise and fall considerably.

applied force, they produce negative external work (work is done on the muscle; see Section 5.2).[12] This stretching of active muscles, attempting to shorten but not producing enough force of their own to overcome the externally applied force, occurs during the decelerating phase of walking. Part of the walking time is spent by muscles producing external work, and part of the walking time is spent by work being done on the muscles. The former is characterized by a positive muscular efficiency and the latter by a negative muscular efficiency. Mean muscular efficiency for the entire act is about zero.[13]

If there were some way of storing mechanical energy at the appropriate points in the walking cycle, it could be recovered to aid in performing other work and muscular efficiency would rise. One way of doing this would be to store energy in an elastic medium. But humans have not developed a very effective elastic medium in the course of their evolution and thus cannot use this mechanism.[14] The energy which is not stored becomes useless heat.

There are other ways of handling the excess of external mechanical energy without elastically storing it. One alternative is to prevent the stretching of active muscles by converting the downward velocity component of the body's center of mass into an upward component later in the walking cycle. This mechanism applies a force to the center of mass at right angles to its direction of motion. When the force is at right angles to the displacement, the muscles that supply the force can neither do work nor have work done on them. The velocity is changed at no expense to muscular work.

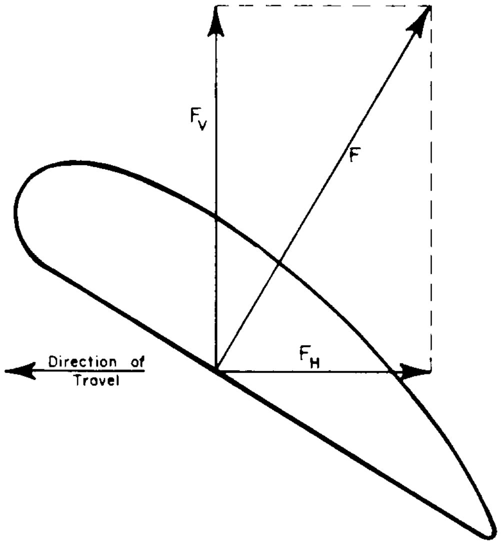

Figure 2.3.3 A wing converts horizontal movement into vertical lift. Here the net force F acting on the wing is decomposed into a horizontal F_H and vertical F_V component.

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[12] Kinesiology is the study of human movement, and biomechanics is the subdiscipline that relates to neuromusculoskeletal aspects of that movement (Winter, 1983). Kinesiologists usually use the term "concentric" contraction for muscles shortening while producing positive external work. They use the term "eccentric" contraction for muscles shortening and producing negative muscular work.

[13] Alexander (1984) compares the changes in energy and speed during walking and running to alternately braking and accelerating while driving a car. The average speed can be held to the same value as during steady driving, but the use of energy in the form of gasoline is much greater this way. The difference lies in the greater dissipation of energy as heat while braking and accelerating, and the resulting efficiency is very low.

[14] This is true for the case of walking. For running, however, there is a considerable amount of elastic energy storage (see Section 2.4).

An example of this is the pole used by the pole vaulter. The vaulter runs at a high speed and thrusts the pole into a box in the ground. As long as the pole is not exactly horizontal, a component is developed in the pole which is perpendicular to the direction of running. This component lifts the vaulter without a vertical component of muscular work required.

A wing also performs this function. A wing is usually thin and tilted (Figure 2.3.3). The tilt enables a vertical force component to be developed from horizontal movement. With wings, the flying animal can change the downward motion of its center of mass into a forward motion without stretching elastic structures or active muscles. Tucker (1975) provides a dramatic example of the benefits of developing this perpendicular force. He considers the results of dropping a pigeon and a rat from a high place. The pigeon merely extends its wings and the perpendicular force changes its motion from vertical to horizontal. The rat, however, must absorb all the developed kinetic energy at the bottom of the fall by stretching elastic structures and active muscles, probably with extremely damaging effect.

Active muscle stretching can also be prevented by precluding the vertical movement of the center of mass of the body. Many fishes achieve this end by balancing the force of gravity with the buoyancy of their swim bladders. Millipedes, with their large number of legs, can support their centers of mass at all times. The extreme of this strategy leads to the wheel. The wheels of a bicycle stabilize the position of the rider's center of mass, and even pedaling while standing up does not result in the stretching of active muscles because when the center of mass falls the motion is translated into horizontal movement. By using external machinery humans can achieve the muscular efficiencies that swimming and flying animals naturally accomplish.

2.4 WALKING AND RUNNING

Moving about by walking and running has been the object of much study. In this section we proceed from the simplest of biomechanical energetic models to theories about control of these processes to experimental correlations of data.

2.4.1 Basic Analysis

Walking is a natural movement in which at least one foot is on the ground at all times (Figure 2.4.1)[15] Because each foot touches the ground for slightly more than half the time, there are stages when both feet are simultaneously on the ground (Alexander, 1984). While stepping, the leg remains nearly straight, and, the position of the center of mass of the body is therefore highest when the leg is vertical and the body passes over the supporting foot. Contrarily, the body is lowest when both feet are touching the ground.

Running is a different mode of locomotion in which each foot is on the ground less than half the time (Figure 2.4.2). There are stages of running during which neither foot is on the ground. The runner travels in a series of leaps, with the center of mass of the body at its highest in midleap. Its lowest point occurs when the trunk passes over the supporting foot, and the supporting leg is bent at this stage. Walking and running are therefore characterized by many dissimilarities, with the major resemblance between the two being forward motion propelled by the legs.

The transition between walking and running occurs at fairly predictable speed of about 2.5 m/sec (6 mi/hr) for normal-sized adults (Alexander, 1984). Why this should be so can be shown easily by a simple model of walking.

As illustrated by Figure 2.4.3, the walker sets a foot on the ground ahead of himself and, while keeping the leg straight, propels himself forward with a speed v. His hipjoint thus moves along an arc of a circle centered on the foot. For purposes of this simple model, the legs will be considered to be sufficiently light that their masses can be ignored compared to

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[15] Except for race walking, where, it has been found, there is a very short time during which neither foot has ground contact.

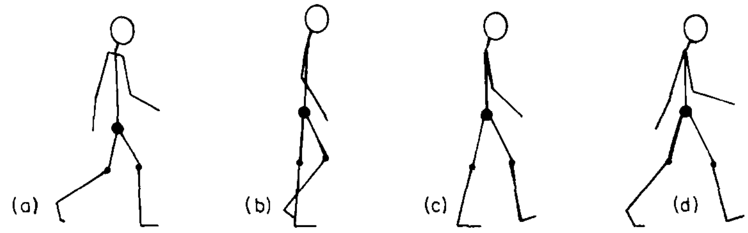

Figure 2.4.1 Four successive stages of a walking stride. In the first stage the trailing foot leaves the ground and the front foot applies a braking force. In the second stage the trailing foot is brought forward off the ground and the supporting foot applies a vertical force. In the third stage the trailing foot provides an acceleration force. In the last stage, both feet are on the ground, with the trailing foot pushing forward and the front foot pushing backward.

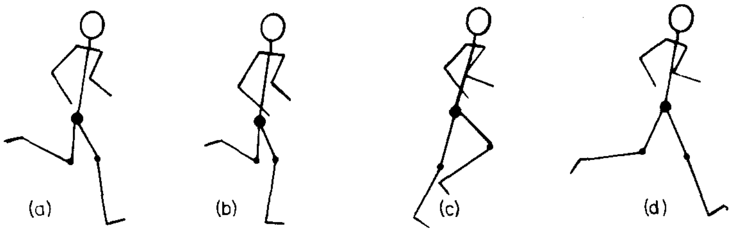

Figure 2.4.2 Four stages of running. Braking and pushing forces are exerted by the feet much as in walking, but much of the otherwise lost energy is stored between the first two stages in the form of elastic strain in the tendons. This energy is then released between the second and fourth stages. During the last stage, no feet are touching the ground; therefore, the opposing forces generated by the feet during the last stage of walking are not present.

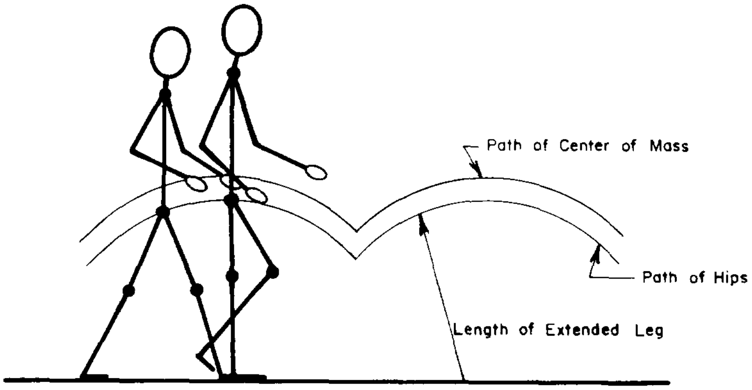

Figure 2.4.3 While walking, the center of mass of the body rises and falls along an arc with a radius depending on the length of the leg. (Adapted and redrawn with permission from Alexander, 1984.)

the trunk, and therefore the body center of mass will occupy a fixed position on the trunk. Hence the center of mass will move along an arc of the same radius as that of the hip joint.

Centripetal force can be calculated from Equation 2.2.26:

$$F_c = \frac{mv^2}{r} \tag{2.2.26}$$

where F_c = centripetal force, N
m = body mass, kg
v = tangential velocity, m/sec
r = arc radius, m

A point moving with speed v along an arc of a circle will have an acceleration toward the center of the circle:

$$\frac{F}{m} = a = \frac{v^2}{r} \tag{2.4.1}$$

where a = acceleration, m/sec^2

When the center of mass is at its highest, this acceleration will be directed vertically downward. Since the walker cannot pull himself downward, his vertical acceleration is limited to the free fall acceleration of gravity:

$$\frac{v^2}{r} \le g \tag{2.4.2a}$$

or

$$v \le \sqrt{gr} \tag{2.4.2b}$$

where g = gravitational acceleration, 9.8 m/sec^2

With a typical leg length of 0.9 m, maximum walking speed is about 3 m/sec, close to the observed 2.5 m/sec in adults. Children, who have shorter legs than adults, break into running at lower forward speeds. These results confirm the analysis resulting in Equation 2.2.34c.

Race walkers exceed this maximum speed, however, traveling about 4 m/sec (Figure 2.4.4). The trick that makes high walking speeds possible is to bend the lower part of the back during walking, thus sticking the pelvis out and lowering the center of mass of the body relative to the hip joint. The center of mass no longer moves in arcs of radius equal to the length of the legs, but in arcs of larger radius. There is less rising and falling, and higher speeds are possible.[16]

More detailed analysis of walking has shown that the simplified approach given previously may be misleading. McMahon (1984) summarizes six movements during walking which modify the gait:

1. *Compass gait*. This is the basic walk characterized by flexions and extensions of the hips and illustrated in Figure 2.4.3. The legs remain stiff and straight.

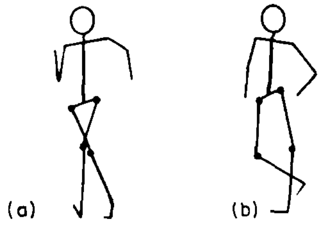

Figure 2.4.4 During race walking the center of mass of the body is kept lower than in ordinary walking by the bending of the back and the tilting of the hips. Because the center of mass rises less, higher speeds are possible compared to ordinary walking.

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[16] It has also been reported that women expend less energy than men walking at the same speed. Presumably this is because of shorter steps taken by women, with consequent smaller fluctuation of vertical height of the pelvis (Booyens and Keatinge, 1957)

2. *Pelvic rotation.* The pelvis rotates around a vertical axis through the center of the body. The amplitude of this rotation is about ± 3° during normal walking speeds and increases at high speeds. The effect of this motion is to increase the effective length of the leg, producing a longer stride and increasing the radius of the arcs of the hip, giving a flatter, smoother movement.
3. *Pelvic tilt.* The pelvis tilts so that the hip on the side with the swinging leg falls lower than the hip on the opposite side. The effect of this movement is to make the trajectory arcs still flatter.
4. *Stance leg knee flexion. By* bending the knee of the leg supporting the weight, the arc is made flatter yet.
5. *Plantar flexion of the stance ankle.* The sole, or plantar surface of the foot, moves down and an ankle of the stance leg flexes just before the toe lifts from the ground. A result of this is that the leg muscles can produce the forces necessary to swing the leg forward during the next phase, but it also results in an effective lengthening of the stance leg during the portion of the arc when the hip is falling. The hip thus falls less than it would without this movement.
6. *Lateral displacement of the pelvis.* The body rocks from side to side during walking, with a lifting of the swing leg.

These motions make walking a much more complex process than the simplified models to this point would suggest. The result of these motions is that walking, although still energy inefficient, is not as inefficient as it would be without them.

Results of calculations of expended power made from respiratory gas measurements are seen in Figure 2.4.5. We have already discussed cycling relative to running, and it is not surprising to see that cyclists expend less energy at any given speed than do runners. Power

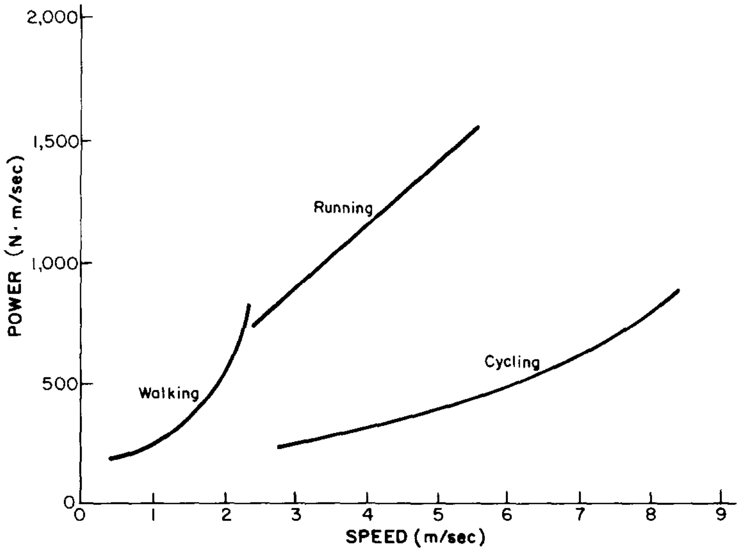

Figure 2.4.5 Power required for walking, running, and cycling by an adult male. Curves for walking and running intersect at about 2.3 m/sec and show that walking is more efficient below the intersection and running is more efficient above. Cycling is more efficient because, presumably, the body does not rise and fall as much as with walking and running. (Redrawn with permission from Alexander, 1984).

required for walking begins at a low value at low speeds (we would expect there to be a minimum in the curve, based the discussion of Section 2.2.3) and rises rapidly to moderate power levels at higher speeds.

Running power begins at moderate levels and rises less slowly than walking at yet higher speeds. An intersection of the walking and running curves occurs at about 2.5 m/sec. If walking is continued beyond this speed, there will be a higher expenditure of energy than if the person switched to running. Similarly, if running is begun before 2.5 m/sec, a higher amount of energy will be expended than if the person walked. It appears that the switch from walking to running occurs because of energy considerations. There is a gradual shift in the walking gait to maintain an optimal energy expenditure (Alexander, 1984) until the abrupt changeover to running to again maintain an optimal energy expenditure.

Human running uses less energy than might be expected because of elastic energy storage. Between the first and second stages of a running stride (Figure 2.4.2) the body is both slowing and falling, simultaneously losing kinetic and potential energy (Alexander, 1984). This energy must be restored between the third and fourth stages. If the energy lost was not stored somewhere, the metabolic energy required from the muscles would be about 1.8 times the actual energy consumption for slow running and 3.0 times the actual consumption for fast running (Alexander, 1984).

Energy is stored by elastic deformation of the muscles and tendons. Muscles may be stretched about 3% of their length before they yield and the energy cannot be recovered elastically. Tendons can stretch about 6%) before breaking (Alexander, 1984). Although elastic energy can be stored in each of these, the tendons are probably the most important structures for energy storage. The ligaments and tendons in the soles of the feet and the Achilles tendon are likely the most important site of energy storage during each running step (Alexander, 1984).

Quadripedal animals have one mode of locomotion, besides walking and running, that humans do not: they gallop at high speed. Galloping involves bending movements of the back which briefly store leg kinetic energy fluctuations as elastic energy, contributing to overall efficiency (Alexander, 1988). These animals appear to have two transitional power points, one from walking to running or trotting and another from running to galloping.

2.4.2 Optimal Control of Walking

Walking has long been recognized as one bodily function that appears to have some built~in optimization operating (see also Sections 3.4.3 and 4.3.4). Walking, for instance, appears to occur at a speed that minimizes the rate of energy expenditure of the body. This can be simply shown from the empirical observation that power consumption of walking, as measured by oxygen consumption, depends on walking speed (Dean, 1965: Milsum. 1966):

$$\dot{E}=a+bs^2 \tag{2.4.3}$$

where $\dot{E}$ = rate of energy usage (or power), N·m/sec
a, b = constants, N·m/sec and N·sec/m
s = walking speed, m/sec
Average power per unit speed is

$$\dot{E}/s=a/s+bs \tag{2.4.4}$$

This represents an average power with two components, one linearly increasing and one hyperbolically decreasing (Figure 2.4.6). Minimum average power can be found by taking the derivative of $\dot{E}/s$ and setting the derivative equal to zero:

$$\frac{d}{ds}(\dot{E}/s)=\frac{-a}{s^2}+b=0 \tag{2.4.5}$$

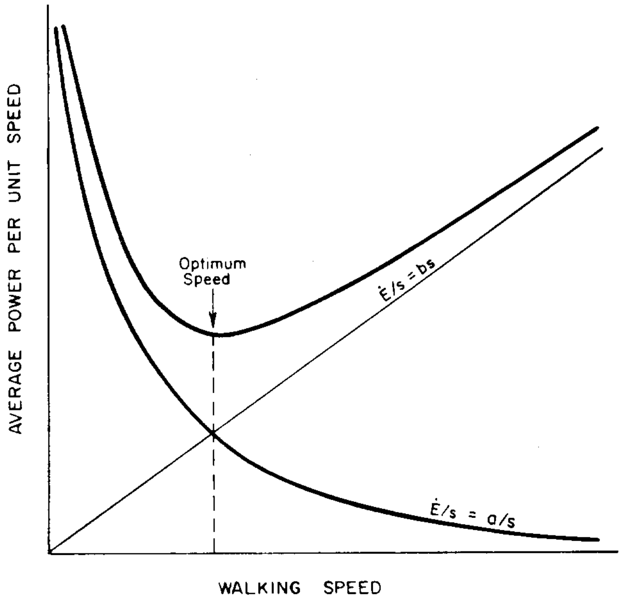

Figure 2.4.6 Walking appears to occur at a speed very close to the optimum based on average power consumption. Two components of average power, one increasing with speed and the other decreasing with speed, make possible a minimum average power.

$$s=\sqrt{a/b} \tag{2.4.6}$$

where s = optimum speed, m/sec

Biochemical models describing walking and running are necessarily complex, not conceptually but parametrically. As noted by Winter (1983), model inputs of muscle electromyographic signals are many for relatively simple movements. Similarly, model outputs of body segment motions, each with 3 degrees of freedom and 15 variables of forces and moments, can soon become overwhelming. It is no wonder, then, that most modelers have greatly oversimplified, constrained, or limited conditions for their models in order to deal with these problems (Onyshko and Winter, 1980; Siegler et al., 1982).

All this is not necessarily bad, however. Depending on the use of the model, a simplified artificial model may be preferred to a complex realistic model. This preference is especially true if the model is to be used to impart understanding of general patterns rather than to diagnostically treat individual malfunctions. Biomechanical models, used to identify causes of abnormal gait patterns or to improve competitive running performance, are necessarily very complex models. Since the objective for including models in this book is to aid general understanding through mathematical description (see Section 1.1), models chosen for inclusion are of the general or simplistic type.

There are many reasons for developing biomechanical models of walking and running. Pierrynowski and Morrison (1985a, b) developed theirs to predict muscular forces; Williams and Cavanagh (1983) and Morton (1985) developed theirs to predict power output during running; Greene's model (1985) has applications to sports; Dul and Johnson (1985) developed a descriptive kinematic model of the ankle; and Hatze and Venter (1981) used their models to investigate the effects of constraints on computational efficiency. Reviews by King (1984) and Winter (1983) summarize many recent modeling attempts.

The one model chosen to be highlighted here uses stepping motion as the object of the model and includes control aspects as well as mechanical descriptions (Flashner et al., 1987). In this model is postulated a hypothetical, but nevertheless plausible, hierarchical structure of stepping control.

The hierarchy of control, diagramed in Figure 2.4.7, includes both open- and closed-loop components. In general, Flashner et al. postulated a system that normally determines, based on previous experience, the trajectory of a step to be taken. This is used as an open-loop procedure during stepping. Only when the controller determines that the trajectory is not proceeding as planned does it take corrective action.

For instance, if an object stands in the way of the step, the leg and foot must move in such a way as to clear the object. Presumably the individual, over the years, has learned how to optimally perform this task, The trajectory that has been stored from previous trials is then used to program hip, leg, and foot motion. The action is quick and sure as long as nothing unexpected happens. The controller samples sensors in the leg to determine if the intended trajectory is being achieved. This sampling may occur at a slower or faster rate, depending on the needs of the controller.

If conditions are found to be not as anticipated (for instance, if heavier shoes are worn or the object moves), feedback control is used to correct the intended action (for instance, muscle forces are increased or activity times are changed). Since feedback control is slower than feedforward control, and feedback control does not effectively use past experience (at least control with constant coefficients, or nonadaptive control), feedback is used only when required.

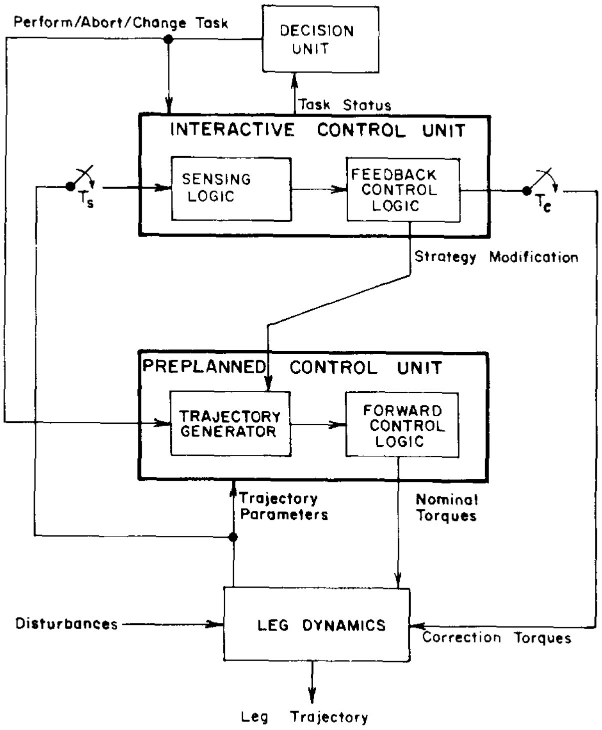

Figure 2.4.7 Hierarchical control of a stepping motion. The decision to perform the motion results in a trial trajectory given to the leg. Sampled data from the leg are sent to the interactive control unit, where it is determined whether or not to send correction torques to the leg to overcome unforeseen disturbances. (Adapted from Flashner et al., 1987.)

The highest level of control decides about task performance: to step or not to step, to change task strategy for feedforward or feedback control units. The interactive control unit is activated only when corrections to the preplanned movement are needed. This is the site of feedback control.

At a lower level comes the preplanned control unit, wherein are stored optimal trajectories parameterized by relevant variables of motion such as step height, step length, and step duration (or, more likely, muscle forces and durations).

The leg dynamics level includes both sensing and activation. Its outputs are used by both the feedforward and feedback control units, and it receives input information from both control units.

During learning the interactive control unit is constantly active. The preplanned control unit determines a candidate trajectory. Joint angles and control torques are calculated and sent to the leg dynamics unit. Some performance criterion is calculated and stored. Over the course of the learning period the task is repeated many times with different candidate trajectories. Each of these yields a different value for the performance index. The trajectory with the most desirable (usually the maximum or minimum) cost or performance index is the trajectory that is remembered.

Performance criteria[17] for the task may include any number of aspects. Minimizing time, energy, peak force, or any combination of these is a possible performance criterion. In addition, there are constraints, such as limits to the force of contact with the ground, velocities, and accelerations. These must be included in the model formulation.

Figure 2.4.8 is a diagram of the dynamic leg model. The leg starts in position 1, fully extended. As it moves to position 2 in midswing the hip and knee are bent. Flashner et al. consider ankle bending only when the foot touches the ground. Hip height decreases but the foot is raised to clear the object. Upon landing, the foot again raises the hip and both hip and knee ankles return to their initial values.

Flashner et al. considered cycloidal velocity profiles for hip and knee joints:

$$\dot{\theta}_H = C_1(\omega - \omega\cos\omega t) \tag{2.4.7a}$$

$$\dot{\theta}_K = C_2(\omega - \omega\cos\omega t) \tag{2.4.7b}$$

where $\dot{\theta}_H$ = time rate of change of hip angle, rad/sec

$\dot{\theta}_K$ = time rate of change of knee angle, rad/sec

ω, C_1, C_2 = constant parameters, whose values are chosen to match experimental data, rad/sec

Cycloids have the advantage that their first and second time derivatives vanish at the beginning ($\omega t = 0$) and end ($\omega t = 2\pi$) of the cycle. Experimental evidence suggests that the smooth transition from a stationary state to a moving state and back again requires both velocity and acceleration to be zero. Cycloids also introduce no more new parameters than do sines or cosines, and, once hip motion is specified, all other angles, positions, velocities, and accelerations are determined.

Kinematic equations can be derived for all segments of the model. Flashner et al. presented these for the foot:

$$X_F = X_H + L_{\mathrm{TH}} \sin\theta_H - L_{\mathrm{SH}} \sin\Theta \tag{2.4.8a}$$

$$Y_F = Y_H - L_{\mathrm{TH}} \sin\theta_H - L_{\mathrm{SH}} \cos\Theta \tag{2.4.8b}$$

$$\dot{X}_F = \dot{X}_H + L_{\mathrm{TH}}\dot{\theta}_H \cos\theta_H - L_{\mathrm{SH}}\dot{\Theta}\cos\Theta \tag{2.4.9a}$$

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[17] Mathematically expressed as a "cost function" or "objective function." See Section 4.3.4.

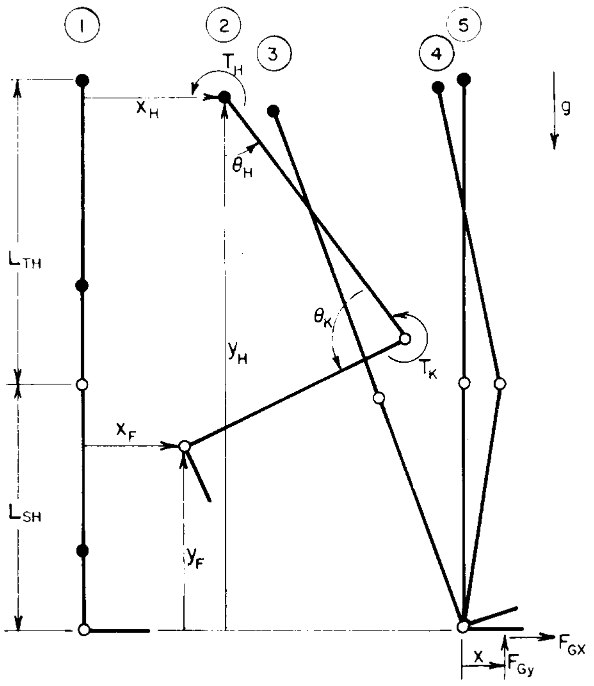

Figure 2.4.8 Schematic representation of the leg dynamic model. Foot and leg motion is not constrained during the swing phase, but once the foot is planted upon landing, leg motion is no longer totally free. Stages represented are (1) begin swing, (2) midswing, (3) begin landing. (4) midlanding, and (5) end landing. (Adapted and redrawn with permission from Flashner et aL, 1987.)

$$\dot{Y}_F = \dot{Y}_H + L_{\mathrm{TH}}\dot{\theta}_H \cos\theta_H - L_{SH}\dot{\Theta}\sin\Theta \qquad (2.4.9b)$$

$$\ddot{X}_F = \ddot{X}_H - L_{TH}\dot{\theta}_H^2 \sin\theta_H + L_{\mathrm{TH}}\ddot{\theta}_H \cos\theta_H + L_{\mathrm{SH}}\dot{\Theta}^2 \sin\Theta - L_{\mathrm{SH}}\ddot{\Theta}\cos\Theta \qquad (2.4.10a)$$

$$\ddot{Y}_F = \ddot{Y}_H + L_{\mathrm{TH}}\dot{\theta}_H^2 \cos\theta_H + L_{TH}\ddot{\theta}_H \sin\theta_H - L_{\mathrm{SH}}\dot{\Theta}^2 \cos\Theta - L_{\mathrm{SH}}\ddot{\Theta}\sin\Theta \qquad (2.4.10b)$$

where X_F, Y_F = position coordinates of the foot in a fixed frame of reference, m
$\dot{X}_F, Y_F$ = velocity components of foot, m
$\ddot{X}_F, Y_F$ = acceleration components of foot, m
L_{TH} = length of the thigh measured from hip joint to knee joint, m
L_{SH} = length of the shank measured from knee joint to ankle, m
θ_H = hip angle, rad

and

$$\Theta = \theta_H + \theta_K \qquad (2.4.11)$$

where θ_K = knee angle, rad

Notice that Equations 2.4.9a, b and 2.4.10a, b are obtained from Equations 2.4.8a, b by simple derivatives.

Flashner et al. also present inverse kinematic equations, that is, equations to predict hip and knee angles and their derivatives from foot position. The reader is referred to Flashner et al. (1987) for these equations.

Model dynamic equations are derived using the Lagrangian method.[18] In generalized form, dynamic equations related to system energy are

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial q_i}\right)-\frac{\partial \mathscr{L}}{\partial q_i}=Q_i+C_i \tag{2.4.12}$$

where $\mathscr{L}$ = system Lagrangian
= difference between system kinetic energy and system potential energy, N·m
q_i = generalized coordinates, m or rad
Q_i = generalized forces, N or N·m
C_i = constraints, N or N·m

For Flashner et al.'s model, the coordinate vector is

$$q = [q_1\, q_2\, q_3\, q_4]^T = [X_H\, Y_H\, \theta_H\, \theta_K]^T \tag{2.4.13}$$

and the force vector is

$$Q = [Q_1 Q_2 Q_3 Q_4]^T = [F_{HX} F_{HY} T_H T_K]^T \tag{2.4.14}$$

where F_{HX}, F_{HY} = force component acting on the hip, N
T_H, T_K = torques acting at hip and knee joints, N·m

The constraint vector is

$$C = J(q)\lambda \tag{2.4.15}$$

where $J(q)$ = Jacobian matrix[19] that relates the Cartesian coordinates of mass centers to generalized coordinates q_i
λ = vector of length to be determined by the system of constraints

During the swing phase of stepping there are no constraints on the motion of the leg. During the landing phase of stepping the motion of the foot is constrained by the surface of the ground. Flashner et al. introduce these constraints by

$$\begin{aligned} X_F &= 0 \\ Y_F &= 0 \end{aligned} \tag{2.4.16}$$

These conditions were used to derive equations of motion for the swing and landing phases.

Flashner et al. (1987) fitted their equations to experimental data as seen in Figure 2.4.9. It can be seen from the figure that the agreement is quite close. Because these data were used as the basis for model calibration, the fit would be expected to be very good as long as the model had general validity.[20] The conclusion which can be reached, therefore, is that the model has sufficient capacity to reproduce reality in very limited circumstances. Whether the model is a good description of the actual control of stepping and whether the model is a good predictor of other data to which it has not been calibrated have yet to be determined.

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[18] Any system of equations which must be solved subject to certain constraints is a candidate for Lagrange's method. Constraints are introduced into the equation set using a series of parameters called Lagrange multipliers. See Appendix 4.1.

[19] The Jacobian matrix contains elements each of which is a partial derivative of a coordinate in one system with respect to a coordinate in another system. All such elements $\partial x_i / \partial q_i$ are included.

[20] Flashner et al. (1987) show closer agreement to their data when the cycloidal motion of the hip is modified using fitting techniques.

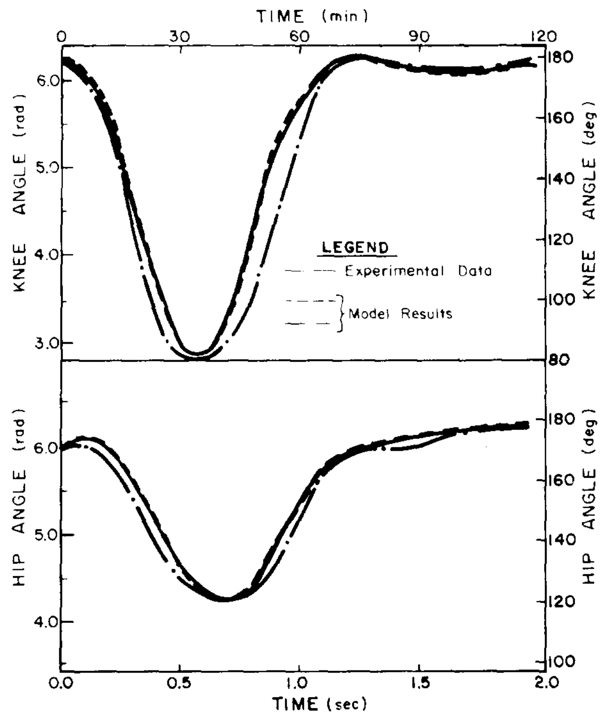

Figure 2.4.9 Comparison of experimental data (solid line) with model results using a cycloidal input (dashed and dashed-dotted lines). (Adapted and redrawn with permission from Flashner et al., 1987).

2.4.3 Experimental Results

Bassett et al. (1985) measured the oxygen cost of running, comparing overground and treadmill running. They found no statistical difference between these two types of running, either on a level surface or on a 5% uphill grade. Level running gave regression lines as follows for treadmill:

$$\dot{V}_{O_2} = 2.22m(10^{-7}s - 10^{-8}), \quad s \le 4.77 \text{ m/sec} \qquad (2.4.17)$$

and for overland running:

$$\dot{V}_{O_2} = 2.02\text{m}\,[10^{-7}s + (2.65 \times 10^{-8})], \quad s \le 4.77 \text{ m/sec} \qquad (2.4.18)$$

where $\dot{V}_{O_2}$ = oxygen consumption, m^3/sec
m = body mass, kg
s = running speed, m/sec

They also reported that the additional oxygen cost caused by air resistance is

$$\Delta\dot{V}_{O_2} = 3.3 \times 10^{-11} s^3 \qquad (2.4.19)$$

where $\dot{V}_{O_2}$ = additional oxygen cost to overcome air resistance, m^3/sec

Measurements of additional metabolic energy used by a runner to overcome air resistance vary widely – from 2% (Bassett et al., 1985) to 16% (Ward-Smith, 1984).

2.5 CARRYING LOADS

Load carrying is an important aspect of manual labor and of certain sports such as weight lifting. Load carrying has been the subject of a great deal of study by exercise physiologists and ergonomicists, and it has even been quantified to a large extent (see Section 5.5). Yet there are quite a few different ways of carrying loads, and quantitative description of these has not been fully completed.

2.5.1 Load Position

Load position has an important effect on the amount of energy required to carry the load. Body weight, for example, is carried with metabolic cost usually less than externally carried loads, since it is reasonably well distributed and its center of mass passes through the center of mass of the body. Light loads carried on the hands, on the head, and high on the back are carried with almost no additional energy penalty except for the weight itself (Table 2.5.1). Heavy loads on the feet and hands, however, pose a muscular burden out of proportion to weight carried (Martin, 1985; Soule and Goldman, 1969)

2.5.2 Lifting and Carrying

Lifting of loads requires an initial isometric muscular contraction to overcome inertia and set the postural muscles followed by a dynamic muscular contraction as the load is moved. The major part of the lift, when it occurs on the job, is composed of dynamic contraction. Pytel and Kamon (1981) studied workers to determine if a simple predictor test could be devised for maximum lifting capacity of an individual. Such a test would be useful in industrial situations. By measuring the dynamic lifting strength of the combined back and arm muscles and comparing this to voluntary maximum acceptable loads lifted, they were able to obtain this simple equation with an r^2 (statistical coefficient of determination) of 0.941:

$$F_m = 295 + 0.66F_{\text{dls}} - 148S_x \tag{2.5.1}$$

where F_m = maximum load to be lifted repetitively, N
F_{dls} = peak force developed during dynamic lifting strength test, N
S_x = sex indicator, 1 for men and 2 for women, dimensionless

TABLE 2.5.1 Relative Energy Cost for Carrying Loads in Different Positions at Different Speeds

	Speed, m/sec		
Condition	67	80	93
No load	1.0	1.0	1.0
Hands, 39 N (4 kg) each	1.0	1.3	1.9
Hands, 69 N (7 kg) each	2.0	1.8	1.8
Head, 13 7 N (14 kg)	1.3	1.0	1.3
Feet, 59 N (6 kg) each	4.2	5.8	6.2
No load, actual energy cost[a]			
$10^8 \cdot [\text{m}^3\ \text{O}_2/\text{sec}]/\text{N}$ body wt	1.79	2.18	2.62
([mL O_2/min]/kg body mass)	(10.5)	(12.8)	(15.4)

Source: Data from Soule and Goldman, 1969.

[a] Wyndam et al. (1963) give a regression equation relating oxygen consumption to work rate of $\dot{V}_{\text{O}_2} = 6.535 \text{ x } 10^{-6} + 2.557 \text{ x } 10^{-7}\,\dot{W}$ where $\dot{V}_{\text{O}_2}$ = oxygen consumption, m^3/sec, and $\dot{W}$ = work rate, N·m/sec (original values of equation coefficients are 0.3921 L/min and $3.467 \text{ x } 10^{-4}$ L/ft·lb). The range of applicability of this equation is not known, however, since data were originally obtained from 88 Bantu tribesmen.

Average values for F_{dls} were 379 N for women and 601 N for men. Maximum dynamic loads were 250 N for women and 544 N for men. Further observations on steelworkers exhibited much more data scattering and much less satisfactory regression equations (Kamon et al., 1982). Goldman (1978) reviewed the field of load lifting and Freivalds et al. (1984) produced a biomechanical model of the load lifting task.

Givoni and Goldman (1971) proposed an empirical equation to predict metabolic energy cost of walking at any given speed and grade while carrying a load. In developing this equation they used data from many different sources but found excellent agreement between data and calculations. They proceeded to provide corrections for load placement (as already discussed), carrying very heavy loads (very heavy loads are carried less efficiently), effect of terrain (higher metabolic costs are involved for rougher walking surface), and running (below a critical speed which depends on external load and grade, running is less efficient than walking). A more thorough presentation of this material can be found in Section 5.5.1, and the reader is referred there.

2.5.3 Using Carts

Using handcarts to carry the load has been found to be much easier than carrying the same load by backpacking. Haisman et al. (1972) tested four commercially available handcarts and found that on a treadmill and on a level asphalt surface a 500 N (50 kg) load required a range of 480-551 N·m/sec to pull while walking. The predicted cost of walking alone was about 446 N·m/sec. The difference between these two was the additional power required to transport the load. Taken a different way, about 800 N-m/sec would have been required to transport the same load on the back.

The same advantage does not appear to hold with rough terrain. Haisman and Goldman (1974) loaded a cart with various weights carefully balanced in the cart. On a blacktop surface little difference was found in metabolic rate of the subject whether he was carrying a 200 N (20 kg) load on his back or 200, 600, or 1000 N loads in a handcart. On a dirt road or dry grass terrain, however, metabolic cost increased up to 50% for the 1000 N load in the cart compared to a 200 N load carried by pack. Although the metabolic cost advantage of carrying loads in a cart is not nearly as great over rough terrain as it is over a smooth surface, carts still make possible transporting loads that would not be possible to carry by hand.

2.6 SUSTAINED WORK

The capacity to perform physical work depends on age, gender, and muscle fiber composition (Kamon, 1981). The demands which any given task make on the body can therefore best be studied by standardizing, or normalizing, to maximal body capacity. For dynamic work efforts it is usually maximum oxygen uptake which is considered to be a measure of maximum capacity (see Section 1.3.4). For static work efforts the maximum muscle force or torque that can be developed by the muscles is useful as a measure of maximum capacity. Thus work can be sustained for periods of time depending on the type (static or dynamic) of work and the fraction of maximum oxygen uptake (dynamic work) or maximum voluntary contraction (static work).

Kamon (1981) gives the maximum time to exhaustion for dynamic work as

$$t_{\text{exh}} = 7200\left(\frac{\dot{V}_{\text{O}_2\,\text{max}}}{\dot{V}_{\text{O}_2}}\right) - 7020 \tag{2.6.1}$$

where t_{exh} = time to exhaustion for sustained dynamic work, sec

$\dot{V}_{\text{O}_2}$ = oxygen uptake, m^3/sec

$\dot{V}_{\text{O}_2\text{max}}$ = maximum oxygen uptake, m^3/sec

Maximum oxygen uptake values maybe found in Table 1.3.2. Kamon recommends working periods of $t_{\text{exh}}/3$ for sustained industrial work involving moving tasks.

Recovery time is generally exponentially related to work intensity. Kamon (1981) suggests that a recovery time of twice the working time is sufficient to replenish ATP in the muscles.

Steady-state (or sustained) dynamic submaximal work at rates above 50% of $\dot{V}_{\text{O}_2\text{max}}$ is accompanied by lactic acid production (see Section 1.3). Duration of resting periods for work rates above 50% $\dot{V}_{\text{O}_2\text{max}}$ should be based on the rate at which lactic acid appears in the blood and the rate at which it disappears. Lactic acid appearance in the blood peaks about 240–300 sec (4–5 min) after muscular exercise ceases, and its elimination rate is linear at about 0.05 mg %/sec (Kamon, 1981). From these, Kamon makes the following recommendation for rest times when working above 50% $\dot{V}_{\text{O}_2\text{max}}$:

$$t_{\text{rest}} = 528\ln\left[\left(\frac{\dot{V}_{\text{O}_2}}{\dot{V}_{\text{O}_2\,\text{max}}}\right) - 0.5\right] + 1476 \tag{2.6.2}$$

where t_{rest} = resting time, sec

Static work can be sustained for a period of time related to maximal voluntary contraction (Kamon, 1981):

$$t_{\text{exh}} = 11.40\left(\frac{\text{MT}_{\text{max}}}{\text{MT}}\right)^{2.42} \tag{2.6.3}$$

where t_{exh} = time to exhaustion for sustained static work, sec
MT = muscle torque, N·m
MT_{max} = maximal muscle torque, N·m

Values for maximal muscle torque may be found in Table 2.2.2.

Rest times for static work are recommended by Kamon (1981):

$$t_{\text{rest}} = 1080\left(\frac{t}{t_{\text{exh}}}\right)^{1.4}\left(\frac{\text{MT}}{\text{MT}_{\text{max}}} - 0.15\right)^{0.5} \tag{2.6.4}$$

where t = time of sustained contraction, sec
t_{exh} = time calculated from Equation 2.6.3, sec

SYMBOLS

A	area, m^2
a	acceleration, m/sec^2
a	constant, N·m/sec
b	constant, N·sec/m
CT	cost of transport, dimensionless
C_i	constraints, N or N·m
c	depth of crouch, m
c_1, c_2	constants, rad/sec
D	distance, m
d_F	distance from the fulcrum to the point of application of a force, m
d_W	distance from the fulcrum to the point of attachment of the load, m
E	energy N·m
E_r	rotational energy, N·m
$\dot{E}$	power, N·m/sec

F	force, N
F_c	centrifugal force, N
F_{cp}	centripetal force, N
F_{dls}	dynamic lifting peak force, N
F_{HX}	force at hip in horizontal direction, N
F_{HY}	force at hip in vertical direction, N
F_m	maximum force, N
F_n	normal force, N
g	acceleration due to gravity, 9.8 m/sec^2
h	height, m
h_m	height of jump on the moon, m
I	moment of inertia, N·m·sec^2
$J(q)$	Jacobian matrix
K	spring constant, N/m
L	length, m
L_F	distance through which the load moves, m
L_{SH}	shank length, m
L_{TH}	thigh length, m
L_W	distance through which the force moves, m
ΔL	change in length, m
$\mathscr{L}$	system Lagrangian
MT	muscle torque, N·m
$\mathrm{MT}_{\mathrm{max}}$	maximum muscle torque, N·m
m	mass, kg
mv	translational momentum, kg·m/sec
P_d	power to overcome drag, N·m/sec
P_i	input power, N·m/sec
P_{nm}	nonmuscular power, N·m/sec
P_{spt}	power to support body weight, N·m/sec
P_w	power to produce external work, N·m/sec
Q_i	generalized forces, N or N·m
q_i	generalized system coordinates, m or rad
r	radius, m
s	speed, m/sec
s_0	initial speed, m/sec
s_F	speed of force movement, m/sec
s_W	speed of load movement, m/sec
S_x	sex indicator, dimensionless
T	period of oscillation, sec
T	vectorial torque, N·m
t	time sec
t_{exp}	time to exhaustion, sec
t_{rest}	resting time sec
$\dot{V}_{\mathrm{O}_2}$	oxygen utilization, m^3/sec
$\dot{V}_{\mathrm{O}_2\,\mathrm{max}}$	maximum oxygen uptake, m^3/sec
v	velocity, m/sec
W	weight, N
W_m	weight of person on the moon, N
X_F	horizontal foot position, m
$\dot{X}_F$	horizontal foot velocity, m
$\ddot{X}_F$	horizontal foot acceleration, m
Y	Young's modulus, N/m^2

Y_F	vertical foot position, m
$\dot{Y}_F$	vertical foot velocity, m
$\ddot{Y}_F$	vertical foot acceleration, m
ε	strain, m/m
η	muscular efficiency, dimensionless
Θ	angular difference, rad
θ	angle, rad
θ_H	hip angle, rad
θ_K	knee angle, rad
$\dot{\theta}$	time rate of change of angle, rad/sec
λ	undetermined vector
μ	friction coefficient, dimensionless
σ	stress, N/m^2
$\sigma_{\max}$	maximum breaking stress, N/ m^2
ϕ	angle of inclination, rad
ω	angular velocity, rad/sec

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